Exam Introduction to Logic (CS & MA)

November 6th, 2014

- ☞ The exam consists of 9 exercises.
- ☞ Write only your student number at the top of the exam. Also put your number at the top of any additional pages.
- ☞ Use a blue or black pen (so no pencils, red pen or marker).
- ☞ Don't forget to fill out and hand in the anonymous evaluation.
- ☞ Only hand in your definite answers. You can take the exam questions and any drafts home.

Good luck!

1: translating propositional logic (13 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

a. We're only going to that zoo if there are no spiders nor snakes.

b. You can feed the sharks, although they won't die if you don't.

2: translating first-order logic (13 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key. The domain of discourse is the set of all humans.

a. John only knows mathematicians who like a computer scientist.

b. Every computer scientist knows a philosopher who has never met a mathematician.

3: formal proofs (28 points) Give formal proofs of the following inferences. Do not forget the justifications.

a.
$$\begin{array}{|l} \forall x(\exists y \neg R(x,y) \to P(x)) \\ \forall x \forall y(x = y \to \neg R(x,y)) \\ \hline P(a) \end{array}$$

b.
$$\begin{array}{|l} \forall x \forall y \forall z((R(x,y) \land R(x,z)) \to R(y,z)) \\ \forall x R(x,x) \\ \hline \forall x \forall y(R(x,y) \to R(y,x)) \end{array}$$

4: truth tables (17 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers.

a. Check with a truth table whether the following formulas are tautologically equivalent.

   (i) $A \leftrightarrow (\neg B \rightarrow \neg C)$

   (ii) $(A \wedge (C \rightarrow B)) \vee (\neg A \wedge C \wedge \neg B)$

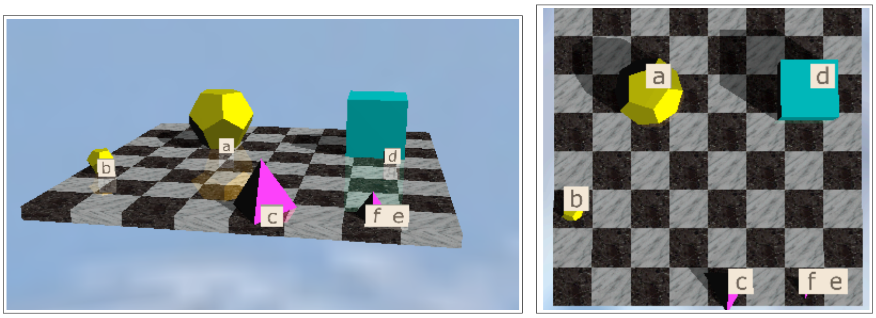
b. Check with a truth table whether the following formula is a tautology.

   $((A \rightarrow B) \vee (A \leftrightarrow B)) \wedge (\neg A \vee B)$

c. Check with a truth table whether the conclusion is a logical consequence of the premises for the following argument. Indicate clearly which rows are spurious.

$$\begin{array}{|l} \mathsf{a = b \wedge (Tet(a) \vee Cube(b))} \\ \mathsf{Tet(a) \leftrightarrow \neg Cube(b)} \\ \hline \mathsf{a = b \rightarrow (Tet(a) \vee \neg Cube(b))} \end{array}$$

5: Tarski's World (29 points)



In the world displayed above a and d are large, c is medium and the other objects are small.

a. In the world displayed above there is only one cube. How can you express this with one formula in the language of Tarski's World such that the formula would be true in every world with only one cube, and false if there were not only one cube?

b. Indicate of each formula below, whether it is true or false in the world displayed above. You do not need to explain your answers.

   (i) $\mathsf{RightOf(a, c) \vee LeftOf(d, b)}$

   (ii) $\mathsf{\forall x(Tet(x) \rightarrow \forall y(Dodec(y) \rightarrow FrontOf(x, y)))}$

   (iii) $\mathsf{\forall x(\neg Dodec(x) \rightarrow \exists y LeftOf(x, y))}$

   (iv) $\mathsf{\neg \exists x \exists y \exists z Between(x, y, z)}$

   (v) $\mathsf{\forall x \exists y \exists z(SameSize(x, y) \rightarrow \neg SameShape(x, y))}$

   (vi) $\mathsf{\forall x \exists y \forall z((Dodec(x) \vee \neg SameSize(y, z)) \leftrightarrow Larger(x, z))}$

   (vii) $\mathsf{\exists x \forall y \exists z((SameRow(y, z) \wedge BackOf(z, x)) \rightarrow SameSize(x, z))}$

c. Explain how the formula below can be made true by removing one object from the world displayed above.

$$\mathsf{\forall x(\exists y(SameCol(x, y) \wedge x \neq y) \rightarrow \exists y \exists z Between(x, y, z))}$$

6: Normal forms propositional logic (5 points)

a. Provide a conjunction normal form (CNF) of the following formula. Show all of the intermediate steps.

   $\neg(A \vee (B \wedge C)) \vee (A \wedge B)$

7: Normal forms first-order logic (15 points)

a. Provide a Skolem normal form of the following formula. Show all of the intermediate steps.

   $\exists x \forall y R(x, y) \rightarrow \forall x \exists y Q(y, x)$

b. Check the satisfiability of the Horn sentence below using the Horn algorithm. If you prefer the conditional form, you may also use the satisfiability algorithm for conditional Horn sentences.

$$(A \vee \neg B) \wedge (\neg A \vee B) \wedge (\neg C \vee \neg D \vee B) \wedge A \wedge (\neg A \vee C)$$

8: Translating function symbols (15 points) Translation the following sentences using the translation key provided. The domain of discourse is the set of all humans.

mostlike(x): the person most like x

Loves(x, y): x loves y

a: Agnes
b: Bailey

a. Not everyone who loves the person most like themselves, loves themselves.

b. Everyone loves the person who is most like Agnes, except Bailey.

c. Unless someone who loves Agnes, loves everyone who is most like Agnes, Bailey does not love Agnes.

9: Semantics (15 points) Let a model $\mathfrak{M}$ with domain $\mathfrak{M}(\forall) = \{1, 2, 3\}$ be given such that

- $\mathfrak{M}(a) = 1$,
- $\mathfrak{M}(b) = 3$,
- $\mathfrak{M}(c) = 2$,
- $\mathfrak{M}(P) = \{1, 3\}$
- $\mathfrak{M}(Q) = \{2, 3\}$
- $\mathfrak{M}(R) = \{\langle 1, 3\rangle, \langle 2, 2\rangle, \langle 2, 1\rangle, \langle 3, 3\rangle\}$

Let h be an assignment such that $h(x) = 2$, $h(y) = 3$, en $h(z) = 2$.

Evaluate the following statements. Follow the truth definition step by step.

a. $\mathfrak{M} \models Q(a) \rightarrow (Q(b) \vee R(y, x))[h]$

b. $\mathfrak{M} \models \forall x(Q(x) \rightarrow R(z, x))[h]$

c. $\mathfrak{M} \models \exists x \exists y(\neg P(x) \rightarrow (Q(x, y) \wedge R(y, y)))[h]$